MEMSENSE

RANDOM DRIFT IN THE MEMSENSE PRODUCT LINE

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MEMSense

Drift in Inertial Sensors

When a value is quoted for drift in a gyroscopic inertial senor we are referring to the random fluctuations in bias that may not be removed by any correction schemes rooted in characterization of deterministic changes in bias, e.g. temperature bias dependence, cross sensitivity. Confusing the discussion of drift is the tendency of professionals referring to deterministic bias changes as “drift” when speaking in general. Our discussion here will refer to the *random* fluctuations by the use of the term drift.

Random Walk

Consider a clumsy man who can walk forwards or backwards along a straight sidewalk. He stumbles either forward or backwards with some probability assigned to forward and backward motion:

probability of moving one step forward, p **Eq. 1**

probability of moving one step backwards $q = 1 - p$ **Eq. 2**

His motion describes a path in space (one dimension) which is called a random walk. A key element that we demand of this process is that each step occurs according to the probability of either forward or backward motion, and therefore an individual step does not depend on the past history of steps.

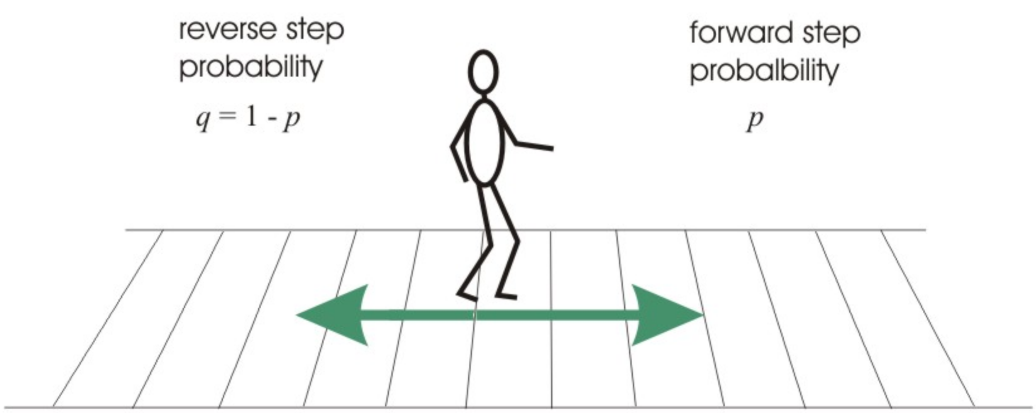

Figure 1: A Random Walker

Lets look at the results (Figure 2) of a single experiment where a man has an equal chance of taking a step forward as he does to take a step backwards ($p = q = 0.5$) and then proceeds to take a set number of steps.

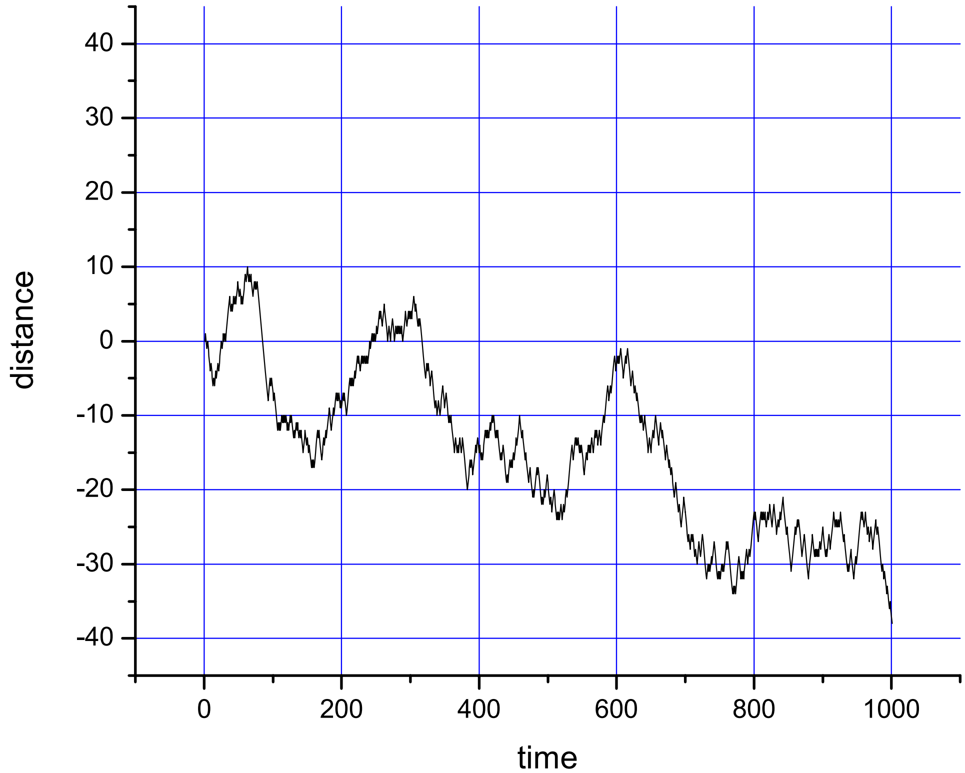

Figure 2: A Single Random Walk Trial

This result may defy intuition upon first examination as it seems reasonable that if you have an equal chance of taking a step forward as backwards that the gain in distance should average to zero (our starting point for the walk) over time. This is clearly not true for this single case. Nevertheless this type of behavior has been confirmed by countless observations and experiments: when a single experiment is conducted, a clear trend away from the starting point is usually observed. Only the average of a large number of separate trials moves towards a displacement of zero (and for a infinite number of trials, exactly zero). We can demonstrate this fact by considering a larger ensemble (N = 20) of walking men (Figure 3). We can clearly see that the average of even twenty trials tends towards zero. Most users are not interested in the fact that the average of many sensors will average to zero. What we want to know is how far we will be from the origin on average with a single observer (a single sensor) after a certain amount of time has elapsed. The random walk can be quantified in terms of the expectation value of position, which is to say the absolute distance we find the walker to be from the origin after a certain number of steps. This value grows with time grows at a rate proportional to the root of the number of steps or $N^{½}$. After 1000 steps, we will find the walker about 32 steps in either direction from the origin.

The bias in gyroscopic sensors fluctuates about a reference value in the same manner encountered with our random walker and the degree to which the bias has drifted can be described in these terms.

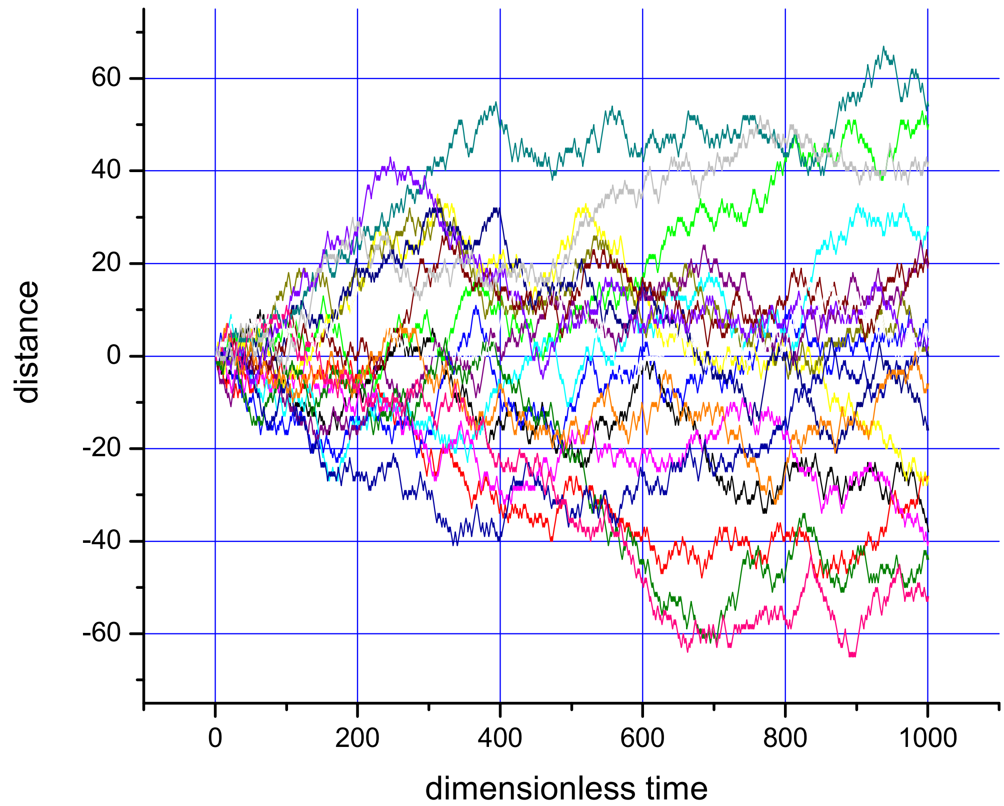

Figure 3: An Ensemble of Random Walk Trials

Quantifying Random Walk

In an inertial sensor system we measure a signal from a angular rate gyroscope that progresses with time. Each step corresponds to a uniform unit of time, Δt, defined by the sample rate. The signal behaves according to an equation of the form.

$$R = Qt^{1/2} \qquad \textbf{Eq. 3}$$

The quantity R is the component of drift due to random fluctuations in bias that behave as a random walk and is measured in degrees per unit time. Often in gyroscopic sensor applications the coefficient Q is used to specify Angle Random Walk (ARW) since it describes the random drift in terms of a convenient constant for noise proportional to $t^{1/2}$ (while simultaneously introduced an unusual unit of measure).

Inertial Sensor Output

The behavior seen in random walk phenomena has applicability to inertial sensor technology in the description of sensor performance. Gyroscopes especially have characteristics that can be quantified in terms of the random walk model.

Random Bias Offset: Bias Instability

The minimum drift in the rate signals is called Bias Instability. Its represents the lowest possible drift attainable in the system under test.

Angular Displacement Derived from Rate Signals: Angle Random Walk

In applications where angle calculations are determined from the rate gyro signals (Figure 4) we observe distinct random walk behavior(Figure 5).

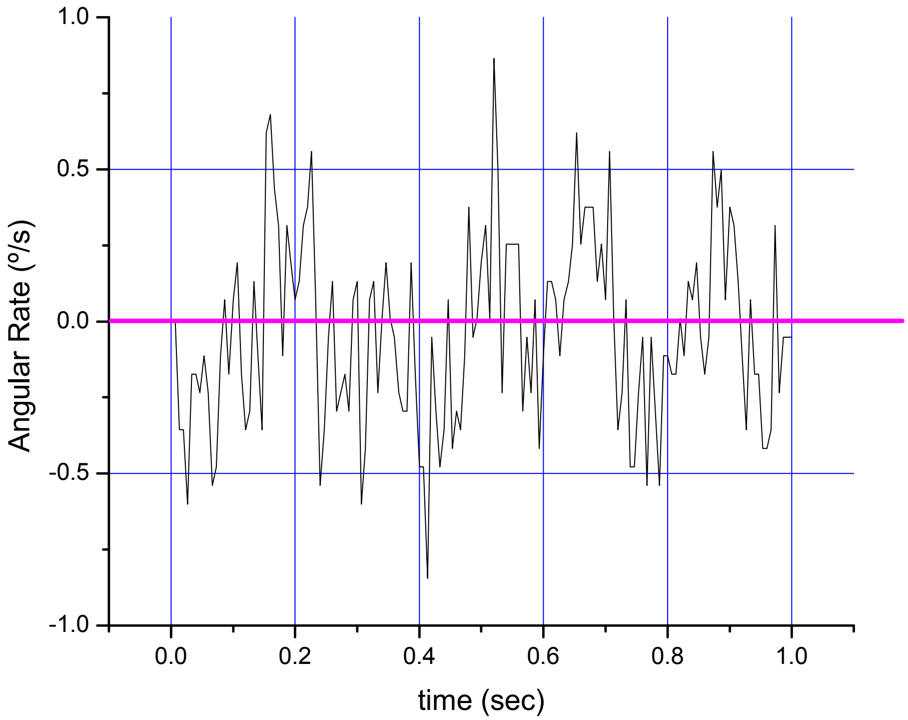

Figure 4: Rate Data

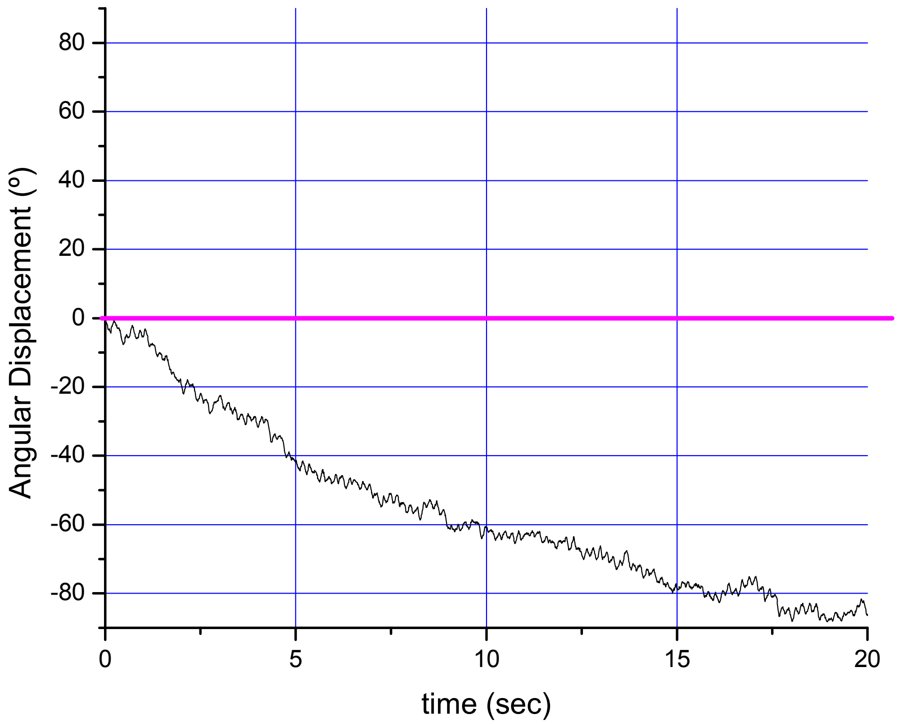

Figure 5: Angular Displacement via Numerical Integration of a MEMS Gyroscope

Long Term Changes in Rate Signal Drift: Rate Random Walk

The output of gyroscopes display changes in bias over long term use that can be characterized by random walk. However, the effects of this noise component are often found to be minimal for gyroscope applications that involve dynamic systems operating over time periods ranging from one second to one hour. Long term changes to bias offset (i.e. long term shelving) will be *randomly distribute*d and may be *permanent* in nature. Even though the drift of an individual sensor can not be predicted, the time scale over which the changes occur can be defined by the RRW and introduces the opportunity to plan for recalibration in critical applications that require extended shelf life.

Determining the Random Noise Components: Allan Deviation

While a researcher at NIST, Dr. David Allan developed a method for quantifying random noise components in oscillators used in clock applications. The various noise components contributing to deviation as a function of averaging time can be easily seen by way of this analysis. The deviation in the signal is calculated for a series of varying averaging times and plotted via increasing averaging time, τ.

In general, the analysis involves collecting zero-rate data for a time period ten times longer than the most significant averaging time (time period of interest) which in most MEMS gyro applications is ranges from one second to one hour† .

The results of a typical Allan deviation analysis:

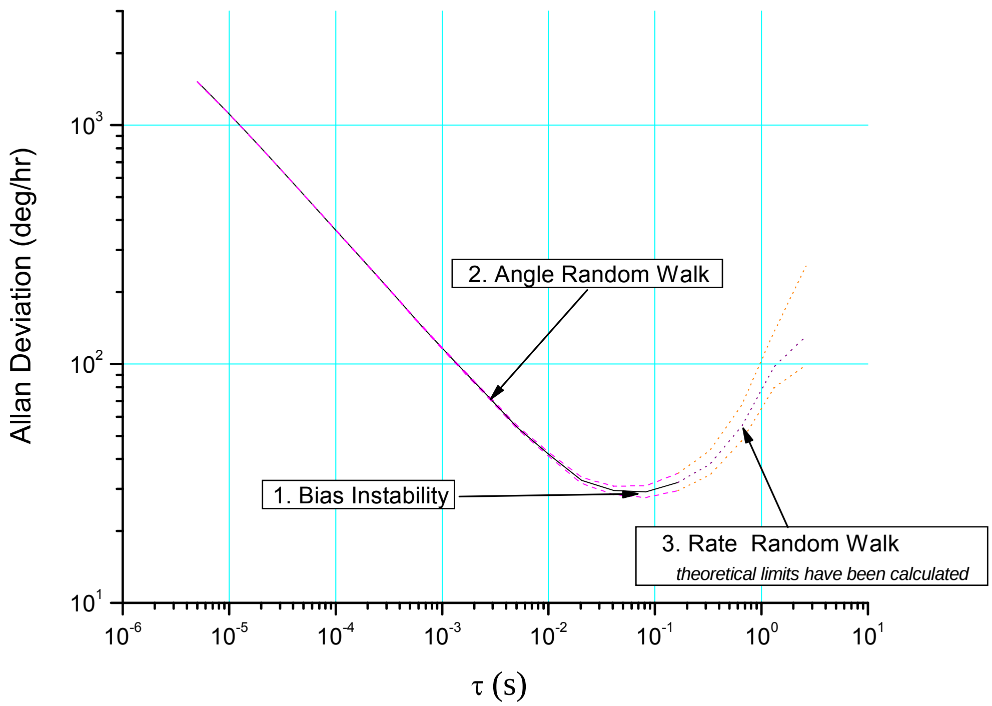

Figure 6: A Typical MEMSense Allan Deviation Plot

Allan Deviation reveals three observable random noise components significant to our gyroscopes: Bias Instability, Angle Random Walk, and Rate Random Walk.

The Allan deviation plot via its representation in log space reveals these noise components by their signature power dependence and corresponding proportionality coefficients:

1. Bias Instability, *B*, is defined as the minimum on this curve for industrial applications.

† Random fluctuations are also observed in longer time periods which are relevant to long term bias, e.g. bias repeatability.

2. Angle Random Walk appears as the negative slope (*m*=-½) section on the left side of the plot.

3. Rate Random Walk corresponds to upwards sloping (*m*=+½) sections that appear on the right side of the plot.

MEMSense Products

	Bias Instability, *B*	Angle Random Walk, *Q*	Rate Random Walk, *K* ‡
Legacy Systems	42 º/hr	3.1º/√hr	n/a
Mid-2009 Onwards	20º/hr	2.29 º/√hr	$\doteq$9.9º/hr$^{3/2}$

Table 1: Random Noise Coefficients in the MEMSense Product Line

‡The RRW number quoted here represents the limit on this quantity established to date.

Deviation due to Angle Random Walk is determined for a time period, *t*, via:

$$\sigma_Q(t) = Qt^{1/2}$$

The contribution from Rate Random Walk may be estimated by using:

$$\sigma_K(t) = \frac{K}{\sqrt{3}} t^{1/2}$$

It should be noted that there are long term random changes in bias that occur on the order of months. These bias vintages behave according to Rate Random Walk. A selection of ten devices display RRW behavior as seen in Figure 7.

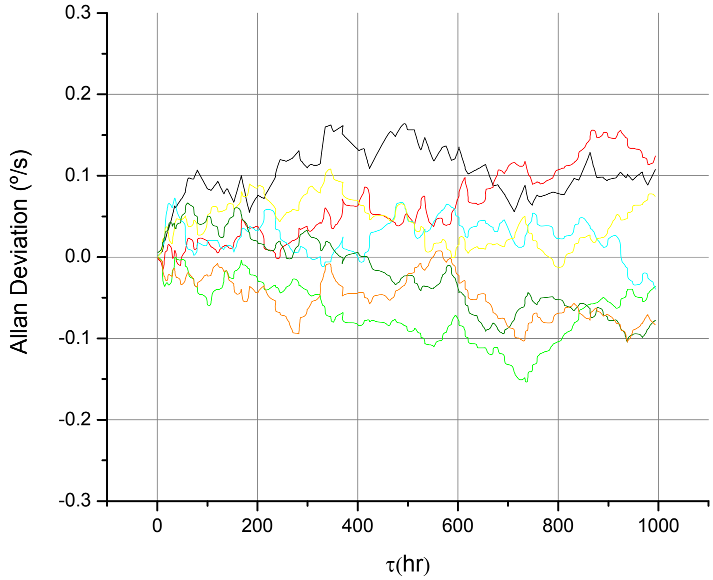

Figure 7: Rate Random Walk

Concluding Remarks

It is important to maintain good perspective of the larger picture here in our discussion of random noise. This process is important in order to characterize the random noise, but keep in mind that there are numerous noise sources that vary in the applicability of a particular gyroscope. Deterministic behavior often outweighs the detrimental effects of the random fluctuations for a particular application. The goal here is to **find the limits** on applicability imposed by the random fluctuations that can not be directly removed with compensation.